

Figure C.2.2 Damage function for pH and pOH

Therefore

$$CO_{iJ} = a_{iJ} 10^{-pJ} + b_{iJ} \quad (C.2.22a)$$

where

$$a_{iJ} = QS_1 / (QU_1 + QS_1) \quad (C.2.22b)$$

$$b_{iJ} = CU_{iJ} \left(\frac{QU_1}{QU_1 + QS_1} \right) \quad (C.2.22c)$$

Expected Damage

The expected damage can be separated into two parts--the damage due to H^+ ions and the damage due to OH^- ions. Let q_i be the density function for pH for the i^{th} source. Define

$$\begin{aligned} \phi_{iH}(x) &= \begin{cases} q_i(x) & ; \quad x \leq 7 \\ 0 & ; \quad x \geq 7 \end{cases} \\ \phi_{iOH}(x) &= \begin{cases} q_i(14-x) & ; \quad x \leq 7 \\ 0 & ; \quad x \geq 7 \end{cases} \end{aligned} \quad (C.2.23)$$

Then the damage due to J ions (where $J = OH^-$ or H^+) is

$$D_{iJ} = \int_0^7 D_J(CO_{iJ}) \phi_{iJ}(pJ) dpJ \quad (C.2.24)$$

and the total damage associated with pH is $D_{iH} + D_{iOH}$.

As discussed earlier, the density function can, depending on the input data, be described by a mean value and either one or two values

of standard deviation (see Figure C.2.1 for an example of the latter distribution). First consider the case when the distribution is described by a single standard deviation. Using (C.2.22a), (C.2.24) becomes

$$D_{1J} = \int_0^7 D_J(a_{1J} 10^{-pJ} + b_{1J}) \phi_{1J}(pJ) dpJ \quad (C.2.25)$$

where ϕ_{1J} is a normal random density function with mean μ_{1J} and standard deviation σ_{1J} . Making a change of variables, let $w = 10^{-pJ}$; then

$$D_{1J} = \int_{10^{-7}}^1 D_J(a_{1J} w + b_{1J}) g_{1J}(w) dw \quad (C.2.26)$$

where g_{1J} is a lognormal density function with corresponding normal distribution having mean $-\mu_{1J}$ and standard deviation σ_{1J} . Analogous to the derivation of (C.2.7) we obtain

$$D_{1J} = \sum_{k=1}^{11} \int_{\alpha_{1Jk}}^{\beta_{1Jk}} \{e_{1Jk} w + f_{1Jk}\} g_{1J}(w) dw \quad (C.2.27)$$

where

$$\alpha_{1Jk} = \frac{d_J(k) - b_{1J}}{a_{1J}} \quad ; \quad k = 1, 2, \dots, 11 \quad (C.2.27a)$$

$$\beta_{1Jk} = \begin{cases} (d_J(k+1) - b_{1J})/a_{1J} & ; \quad k = 1, 2, \dots, 10 \\ \infty & ; \quad k = 11 \end{cases} \quad (C.2.27b)$$

If $\alpha_{1Jk} < 10^{-7}$, then reset $\alpha_{1Jk} = 10^{-7}$ and $\beta_{1J(k-1)} = 10^{-7}$. If $\beta_{1Jk} > 1$, then reset $\beta_{1Jk} = 1$ and $\alpha_{1J(k+1)} = 1$. Also

$$e_{iJk} = \begin{cases} a_i / (d_J(k+1) - d_J(k)) & ; \quad k=1, 2, \dots, 10 \\ 0 & ; \quad k=11 \end{cases} \quad (C.2.27c)$$

$$f_{iJk} = \begin{cases} \frac{(b_{iJ} - d_J(k))}{(d_J(k+1) - d_J(k))} + (k-1) & ; \quad k=1, 2, \dots, 10 \\ 10 & ; \quad k=11 \end{cases} \quad (C.2.27d)$$

(C.2.27) can be rewritten

$$D_{iJ} = \sum_{k=1}^{11} I_L(e_{iJk}, f_{iJk}, \alpha_{iJk}, \beta_{iJk}, -\mu_{iJ}, \sigma_{iJ}) \quad (C.2.28)$$

where I_L is defined in Section C.4.

Now consider the case when the density function is of the form shown in Figure C.2.1 (i.e., the density is defined in terms of a mean μ_{iH} and two standard deviations σ_{iH} and σ_{iOH}). Then the density function for pH can be written

$$\begin{aligned} q_i(x) &= q_i(x) \phi(-\infty, \mu_{iH}, x) + q_i(x) \phi(\mu_{iH}, \infty, x) \\ &= q_{iH}(x) + q_{iOH}(x) \end{aligned} \quad (C.2.29)$$

where ϕ is the characteristic function defined in (C.2.2). $q_{iH}(x)$ is the result when a normal density function with mean μ_{iH} and standard deviation σ_{iH} is restricted to the range $x \leq \mu_{iH}$ and set to zero for $x > \mu_{iH}$. q_{iOH} is similarly defined.

There are two cases to consider: $\mu_{iH} \leq 7$ or $\mu_{iH} > 7$. First suppose $\mu_{iH} > 7$, then using (C.2.23)

$$\phi_{iH}(x) = \begin{cases} q_{iH}(x) & ; \quad x \leq 7 \\ 0 & ; \quad x > 7 \end{cases} \quad (C.2.30a)$$

and

$$\phi_{iOH}(x) = \begin{cases} q_{iOH}(14 - x) & ; \quad x < \mu_{iOH} \\ q_{iH}(14 - x) & ; \quad \mu_{iOH} \leq x \leq 7 \\ 0 & ; \quad x > 7 \end{cases} \quad (C.2.30b)$$

For this case, the formula for D_{iH} is given by (C.2.27). D_{iOH} , however, is now (analogous to (C.2.25))

$$\begin{aligned} D_{iOH} &= \int_0^{\mu_{iOH}} D_J(a_{iOH}10^{-pJ} + b_{iOH}) q_{iOH}(14 - pJ) dpJ \\ &\quad + \int_{\mu_{iOH}}^7 D_J(a_{iOH}10^{-pJ} + b_{iOH}) q_{iH}(14 - pJ) dpJ \\ &= \int_{10^{-7}}^{10^{-\mu_{iOH}}} D_J(a_{iOH}w + b_{iOH}) \psi_{iH}(w) dw \\ &\quad + \int_{10^{-\mu_{iOH}}}^1 D_J(a_{iOH}w + b_{iOH}) \psi_{iOH}(w) dw \end{aligned} \quad (C.2.31)$$

where ψ_{iH} and ψ_{iOH} are lognormal density functions whose corresponding normal distributions have mean $-\mu_{iOH}$ and standard deviations σ_{iH} and σ_{iOH} respectively. Using (C.2.19), (C.2.31) becomes

$$\begin{aligned}
D_{iOH} &= \sum_{k=1}^{\ell_i-1} \int_{\alpha_{iOHk}}^{\beta_{iOHk}} \{e_{iOHk} w + b_{iOHk}\} \psi_{iH}(w) dw \\
&\quad + \int_{\alpha_{iOH\ell_i}}^{10^{-\mu_{iOH}}} \{e_{iOH\ell_i} w + b_{iOH\ell_i}\} \psi_{iH}(w) dw \\
&\quad + \int_{10^{-\mu_{iOH}}}^{\beta_{iOH\ell_i}} \{e_{iOH\ell_i} w + b_{iOH\ell_i}\} \psi_{iOH}(w) dw \\
&\quad + \sum_{k=\ell_i+1}^{11} \int_{\alpha_{iOHk}}^{\beta_{iOHk}} \{e_{iOHk} w + b_{iOHk}\} \psi_{iOH}(w) dw \\
&= \sum_{k=1}^{\ell_i-1} I_L(e_{iOHk}, f_{iOHk}, \alpha_{iOHk}, \beta_{iOHk}, -\mu_{iOH}, \sigma_{iH}) \\
&\quad + I_L(e_{iOH\ell_i}, f_{iOH\ell_i}, \alpha_{iOH\ell_i}, 10^{-\mu_{iOH}}, -\mu_{iOH}, \sigma_{iH}) \\
&\quad + I_L(e_{iOH\ell_i}, f_{iOH\ell_i}, 10^{-\mu_{iOH}}, \beta_{iOH\ell_i}, -\mu_{iOH}, \sigma_{iOH}) \\
&\quad + \sum_{k=\ell_i+1}^{11} I_L(e_{iOHk}, f_{iOHk}, \alpha_{iOHk}, \beta_{iOHk}, -\mu_{iOH}, \sigma_{iOH})
\end{aligned} \tag{C.2.32}$$

where $\ell_i \in \{1, 2, \dots, 11\}$ is defined so that $\alpha_{iOH\ell_i} \leq 10^{-\mu_{iOH}} < \beta_{iOH\ell_i}$. Analogously if $\mu_{iH} \leq 7$, then the equation for D_{iOH} is given by (C.2.27), and D_{iH} becomes

$$\begin{aligned}
D_{iH} = & \sum_{k=1}^{\ell_i-1} I_L(e_{iHk}, f_{iHk}, \alpha_{iHk}, \beta_{iHk}, -\mu_{iH}, \sigma_{iOH}) \\
& + I_L(e_{iH\ell_i}, f_{iH\ell_i}, \alpha_{iH\ell_i}, 10^{-\mu_{iH}}, -\mu_{iH}, \sigma_{iOH}) \\
& + I_L(e_{iH\ell_i}, f_{iH\ell_i}, 10^{-\mu_{iH}}, \beta_{iH\ell_i}, -\mu_{iH}, \sigma_{iH}) \\
& + \sum_{k=\ell_i+1}^{11} I_L(e_{iHk}, f_{iHk}, \alpha_{iHk}, \beta_{iHk}, -\mu_{iH}, \sigma_{iH})
\end{aligned}$$

where $\ell_i \in \{1, 2, \dots, 11\}$ satisfies $\alpha_{iH\ell_i} \leq 10^{-\mu_{iH}} \leq \beta_{iH\ell_i}$.

Probability of Violation

The probability that the standard for either pH or pOH, for the i^{th} source, is violated is

$$\rho_{iJ} = \int_{-\infty}^{EFST_{iJ}} \phi_{iJ}(pJ) dpJ \quad (C.2.34)$$

where ϕ_{iJ} is given by (C.2.23).

For the case where the probability distribution is defined by a mean μ_{iH} ($\mu_{iOH} = 14 - \mu_{iH}$) and a single standard deviation $\sigma_{iH} = \sigma_{iOH}$, ρ_{iJ} is given by

$$\rho_{iJ} = I_N(0, 1, -\infty, EFST_{iJ}, \mu_{iJ}, \sigma_{iJ}) \quad (C.2.35)$$

where I_N is defined in Section C.4. The probability of no violation due to both H^+ and OH^- ions is $1 - (\rho_{iH} + \rho_{iOH})$.

Now consider the case where the density function is defined in terms of a mean and two standard deviations σ_{iH} and σ_{iOH} (see Figure C.2.1). Suppose $\mu_{iH} > 7$, then ρ_{iH} is given by (C.2.34). If $\mu_{iOH} > EFST_{iOH}$ then ρ_{iOH} is also given by (C.2.35), otherwise

$$\begin{aligned}\rho_{iOH} &= \int_{-\infty}^{\mu_{iOH}} \phi_{iOH}(x)dx + \int_{\mu_{iOH}}^{EFST_{iOH}} \phi_{iH}(x)dx \\ &= 0.5 + I_N(0, 1, \mu_{iOH}, EFST_{iOH}, \mu_{iOH}, \sigma_{iH})\end{aligned}\quad (C.2.36)$$

Similarly, if $\mu_{iH} \leq 7$, then ρ_{iOH} is given by (C.2.36). If $\mu_{iH} > EFST_{iH}$, then ρ_{iH} is also given by (C.2.35), otherwise

$$\rho_{iH} = 0.5 + I_N(0, 1, \mu_{iH}, EFST_{iH}, \mu_{iH}, \sigma_{iOH})\quad (C.2.37)$$

The probability of no violation is then $1 - (\rho_{iH} + \rho_{iOH})$.

C.2.4 Temperature

The damage due to heat from an effluent is a function of the change in temperature from its ambient value.

Inputs

The data needed to calculate expected damage and the probability of a violation are:

For the i^{th} source:

$$\mu_{iT} = \text{mean temperature change } (^{\circ}\text{C})$$

σ_{iT} = standard deviation of the temperature change ($^{\circ}\text{C}$)

QU_i = flow of stream above source (MI/day)

QS_i = effluent flow (MI/day)

The damage function which is a function of the temperature change from ambient is of the same form as (C.2.1) with the breakpoints $d_T(k)$ given in Table 6.1.

Change Temperature

The temperature change or temperature difference between the influent and effluent is measured for various industries. This section specifies the calculations needed to determine expected damage and probability of a violation.

The temperature downstream from the source is

$$TO_i = \frac{TU_i QU_i + TS_i QS_i}{QU_i + QS_i} \quad (C.2.38)$$

where TS_i is the temperature of the effluent and TU_i is the upstream temperature. The change in the temperature of the river, ΔT_i , is

$$\Delta T_i = TO_i - TU_i = \frac{(TS_i - TU_i) QS_i}{QU_i + QS_i} \quad (C.2.39)$$

Letting $\Delta TS_i = TS_i - TU_i$ be the change between the influent and effluent temperature, we have

$$\begin{aligned} \Delta T_i &= \Delta TS_i \left(\frac{QS_i}{QU_i + QS_i} \right) \\ &= a_{iT} \Delta TS_i \end{aligned} \quad (C.2.40a)$$

where

$$a_{iT} = \frac{QS_i}{QU_i + QS_i} \quad (C.2.40b)$$

Expected Damage

The expected damage due to temperature change is

$$\begin{aligned} D_{iT} &= E(D_T(\Delta T_i)) \\ &= \int D_T(\Delta T_i) \phi_{iT}(\Delta TS) d(\Delta TS) \end{aligned} \quad (C.2.41)$$

where ϕ_{iT} is the probability density function of the change in effluent temperature. Combining (C.2.40) and (C.2.41)

$$D_{iT} = \int D_T(a_i \Delta TS) \phi_{iT}(\Delta TS) d(\Delta TS) \quad (C.2.42)$$

Since the damage function is in the same form as in Section C.2.1 (with $b_i = 0$), the expression for D_{iT} is the same as given by (C.2.7), (C.2.8) with $j = T$.

Probability of a Violation

The probability that the standard for temperature is not violated for source i is given by (C.2.10) with $j = T$.

C.3 EXPECTED DAMAGE AND PROBABILITY OF VIOLATION DERIVATION --SEVERAL SETS OF STANDARDS

This section describes the derivation of the expected damage from a source and the probability of violation when there are several outfalls, each with its own effluent standards. The most complicated case treated occurs when the outfalls flow into different bodies of water.

C.3.1 Inputs

The data needed to calculate expected damage and probability of violation are:

For source i :

$R_{i\ell}$ = index set of outfalls flowing into stream ℓ .

For source i , stream ℓ :

$QU_{i\ell}$ = flow of stream above source (Ml/day)

$DOSAT_{i\ell}$ = saturation level of DO in stream (mg/l)

$K_{BOD-DO_{i\ell}}$ = BOD_5 - DO transfer coefficient

For source i , outfall k :

P_{ik} = index set of pollutants

μ_{ijk} = mean of mass loading of j^{th} pollutant (kg)

σ_{ijk} = standard deviation of mass loading of j^{th} pollutant (kg)

γ_{ijk} = distribution of j^{th} pollutant

QS_{ik} = effluent flow (Ml/day)

$EFST_{ijk}$ = effluent standard for j^{th} pollutant (kg)

CU_{ijk} = upstream concentration of j^{th} pollutant (mg/l)

\bar{u}_{iJk} = mean of p_J , $J = H$ or OH (pH or pOH)

$\bar{\sigma}_{iJk}$ = standard deviation of p_J , $J = H$ or OH (pH or pOH)

$EFST_{iJ}$ = effluent standard of p_J , $J = H$ or OH (pH or pOH)

C.3.2 Expected Damage

Expected Damage on Stream ℓ - All Variables Except pH

The expected damage to a single stream depends on the total mass load of pollutants flowing into it. It is assumed here that the outfalls are located close enough together, as far as damage is concerned, for the effluents to be considered as coming from a single outfall. The following development is for all constituents except pH (pH will be treated later). It is assumed that the distribution of the j^{th} pollutant is the same for all the pipes, that is, $\gamma_{ijk} = \text{constant}$ for $k \in R_{i\ell}$. The effluent flows add, therefore

$$\begin{aligned} QS'_{i\ell} &= \text{total effluent flow from source } i \text{ into stream } \ell \\ &= \sum_{k \in R_{i\ell}} QS_{ik} \end{aligned} \quad (\text{C.3.1})$$

Normal Case - For the case where the probability distribution of the mass loadings is normal, the distribution of the total loading into stream ℓ is normal with mean and variance equal to the sum of the individual means and variances. (This is true under the assumption that the loadings are independent, which will most likely be the case, since different outfalls are almost always connected to different processes.)

Therefore, for source i , stream ℓ :

$$\begin{aligned} \mu'_{ij\ell} &= \text{mean of mass loading of } j^{\text{th}} \text{ pollutant} \\ &= \sum_{k \in R_{i\ell}} \mu_{ijk} \end{aligned} \quad (\text{C.3.2})$$

$$\begin{aligned} (\sigma'_{ij\ell})^2 &= \text{variance of mass loading of } j^{\text{th}} \text{ pollutant} \\ &= \sum_{k \in R_{i\ell}} \sigma_{ijk}^2 \end{aligned} \quad (\text{C.3.3})$$

Lognormal Case - In the lognormal case, the sum of lognormal random variables is no longer lognormal. In fact, the distribution is, in general, very complicated, and to use it would be untractable for our purposes. Since the mean and variance of the distribution of the total mass loading are equal to the sum of the means and variances, respectively, of the individual loadings, an approximation of the distribution can be obtained by assuming that the resulting distribution is lognormal with mean and variance equal to the sum of the means and variances respectively. This will be a very good approximation for the cases of interest.

Recalling that μ_{ijk} and σ_{ijk}^2 are the mean and variance of the corresponding normal distribution, the mean, m_{ijk} , and variance, v_{ijk} , of the lognormal distribution are

$$m_{ijk} = 10^{-(\mu_{ijk} + 1/2 \gamma \sigma_{ijk}^2)} \quad (C.3.4)$$

and

$$v_{ijk} = m_{ijk}^2 \left\{ 10^{\gamma \sigma_{ijk}^2} - 1 \right\} \quad (C.3.5)$$

where $\gamma = \ln 10$. The mean and variance of the total mass loading are then

$$m'_{ijl} = \sum_{k \in R_{il}} m_{ijk} \quad (C.3.6)$$

$$v'_{ijl} = \sum_{k \in R_{il}} v_{ijk} \quad (C.3.7)$$

Assuming that the resulting distribution is lognormal, the corresponding normal distribution has variance and mean

$$(\sigma'_{ijl})^2 = \frac{1}{\gamma} \log \left(\frac{v'_{ijl}}{(m'_{ijl})^2} + 1 \right) \quad (C.3.8)$$

$$\mu'_{ij\ell} = \log m'_{ij\ell} - 1/2 \gamma(\sigma'_{ij\ell})^2 \quad (C.3.9)$$

Expected Damage on Stream ℓ - pH

Since the distribution of pH (or pOH) is normal, the distribution of H^+ (or OH^-) ions is lognormal. Since the loadings of ions add, the distribution of pH for the total effluent into stream ℓ is very similar to the lognormal case just discussed. The major difference arises from the fact that pH is defined as the negative of the log of the concentration of ions. Thus, equations (C.3.4) through (C.3.9) hold with

$$\mu_{iJk} = -\bar{\mu}_{iJk} \text{ obtaining } \mu'_{iJ\ell}, \text{ then } \bar{\mu}'_{iJ\ell} = -\mu'_{iJ\ell}.$$

Total Expected Damage

All the data have now been combined in terms of the total loading due to source i in stream ℓ . The formulas of Section C.2 can now be used to obtain the expected damage where all the variables have an extra subscript denoting the stream into which the outfall flows. So letting $D_{ij\ell}$ be the expected damage due to the flow of the j^{th} constituent from source i into stream ℓ , the expected damage due to the i^{th} source can then be written (analogous to (C.1.2))

$$c_i = \max_{j,\ell} D_{ij\ell} \quad (C.3.10)$$

C.3.3 Probability of Violation

The calculation of the probability of violation is not complicated since, if we assume that the effluents from the various outfalls are independent, the probability of no violation from all the outfalls is the product of the probability of no violation in each of the outfalls. To be precise, let

$$p_{ijk} = \text{probability of no violation due to pollutant } j, \\ \text{outfall } k, \text{ source } i.$$

The calculation of p_{ijk} is discussed in Section C.2. Using (C.1.3) and (C.1.4), the probability of no violation of any standard from outfall k, source i is

$$p_{ik} = \begin{cases} \prod_{j \in P_{ik}} p_{ijk} & ; \quad \text{uncorrelated constituents} \\ \min_{j \in P_{ik}} p_{ijk} & ; \quad \text{correlated constituents} \end{cases} \quad (C.3.11)$$

The probability of no violation from any pollutant of any outfall for the source i is then

$$p_i = \prod_k p_{ik} \quad (C.3.12)$$

where we have assumed that the pollutant loadings in the outfalls are independent.

C.4 CALCULATION OF IMPORTANT INTEGRALS

C.4.1 Normal Case

$$I_N = I_N(a, b, \alpha, \beta, \mu, \sigma) = \int_{\alpha}^{\beta} (ax + b) \tilde{f}(x) dx \quad (C.4.1)$$

where \tilde{f} is normal with mean μ and variance σ^2 . Therefore

$$\begin{aligned} I_N &= \int_{\alpha}^{\beta} \frac{ax + b}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\} dx \\ &= \int_{\alpha}^{\beta} \frac{a(x - \mu) + \mu a + b}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\} dx \quad (C.4.2) \end{aligned}$$

Let $\hat{x} = (x - \mu)/\sigma$, $\hat{\alpha} = (\alpha - \mu)/\sigma$ and $\hat{\beta} = (\beta - \mu)/\sigma$, then

$$\begin{aligned}
I_N &= \frac{1}{\sqrt{2\pi}} \int_{\hat{\alpha}}^{\hat{\beta}} \left(a\hat{x} + \frac{\mu a + b}{\sigma} \right) e^{-\hat{x}^2/2} \sigma d\hat{x} \\
&= a\sigma [f(\hat{\alpha}) - f(\hat{\beta})] + (\mu a + b) [F(\hat{\beta}) - F(\hat{\alpha})]
\end{aligned} \tag{C.4.3}$$

where F and f are, respectively, the standard normal distribution and density function with mean 0 and standard deviation 1.

C.4.2 Lognormal Case

$$I_L = I_L(a, b, \alpha, \beta, \mu, \sigma) = \int_{\alpha}^{\beta} (ay + b) g(y) dy$$

where g is a lognormal density function* whose corresponding normal density function has mean μ and variance σ^2 . Let $\bar{\alpha} = \log \alpha$, $\bar{\beta} = \log \beta$ and $x = \log y$, then

$$I_L = \int_{\bar{\alpha}}^{\bar{\beta}} (ae^{kx} + b) \tilde{f}(x) dx \tag{C.4.4}$$

where \tilde{f} is normal with mean μ , variance σ^2 , and where we used the identity $y = 10^x = e^{kx}$, $k = \ln 10$. Thus

$$I_L = \int_{\bar{\alpha}}^{\bar{\beta}} \frac{(ae^{kx} + b)}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\} dx \tag{C.4.5}$$

Now

$$\begin{aligned}
kx - \frac{(x - \mu)^2}{2\sigma^2} &= \frac{-1}{2\sigma^2} (x^2 - 2x(\mu + \sigma^2 k) + \mu^2) \\
&= -\frac{(x - (\mu + \sigma^2 k))^2}{2\sigma^2} + \frac{(2k\mu + \sigma^2 k^2)}{2}
\end{aligned} \tag{C.4.6}$$

So

* $g(y) = 0$ for $y < 0$, so if α or β is less than 0, set them equal to zero for remaining calculations.

$$\begin{aligned}
I_L &= \int_{\bar{\alpha}}^{\bar{\beta}} \frac{a}{\sqrt{2\pi\sigma^2}} \exp \left\{ k\mu + \frac{\sigma^2 k^2}{2} \right\} \exp \left\{ - \frac{(x - (\mu + \sigma^2 k))^2}{2\sigma^2} \right\} dx \\
&\quad + \int_{\bar{\alpha}}^{\bar{\beta}} \frac{b}{\sqrt{2\pi\sigma^2}} \exp \left\{ - \frac{(x - \mu)^2}{2\sigma^2} \right\} dx \\
&= a \exp \left\{ k\mu + \frac{\sigma^2 k^2}{2} \right\} (F(\beta_2) - F(\alpha_2)) \\
&\quad + b (F(\beta_1) - F(\alpha_1))
\end{aligned} \tag{C.4.7}$$

where

$$\begin{aligned}
\beta_1 &= \frac{\bar{\beta} - \mu}{\sigma} \\
\beta_2 &= \frac{\bar{\beta} - (\mu + \sigma^2 k)}{\sigma} = \beta_1 - \sigma k \\
\alpha_1 &= \frac{\bar{\alpha} - \mu}{\sigma} \\
\alpha_2 &= \frac{\bar{\alpha} - (\mu + \sigma^2 k)}{\sigma} = \alpha_1 - \sigma k
\end{aligned}$$

and F is defined in Section C.4.1.

APPENDIX D

RESOURCES REQUIRED TO MONITOR A SOURCE

The monitoring resources, r_i , required to monitor source i include field, laboratory, office, and transportation costs. The field and laboratory costs contain costs due to manpower and equipment. Each monitoring agency should examine its costs to determine r_i , since these costs will vary due to differences in agency structure, size of regions that are in agencies' jurisdiction, etc. The purpose of this Appendix is to develop reasonable values for r_i to be used in the demonstration part of the project. The transportation costs to travel to the various sites are assumed small and will be neglected.

D.1 FIELD AND OFFICE COSTS

Estimates of manpower requirements for compliance monitoring are given in [D1]. It is estimated that it will take 8 man-days to travel to plant, set up equipment, take measurements, remove equipment, and return to point of origin. If more than 5 outfalls are to be sampled, the manpower requirements must be increased. Also, there may be some savings if additional surveys are conducted in the same vicinity. Mr. R. Christianson of the Michigan Water Resources Commission* estimated that a two man crew can make one 24-hour composite measurement in two days (including set-up and removal) and that the crew can handle four closely spaced outfalls in this period. Combining these estimates, we shall assume that it takes 2 men 2-1/2 days (or 5 man-days) to monitor 4 outfalls. We shall assume that the two man team can be divided between two sources

* Private communication.

if, at most, two outfalls are located at each source. The office cost to process the compliance monitoring data is estimated, in [1], to be 3 man-days. In addition, Mr. Christianson, estimated the equipment costs to monitor the sources at about \$2,500/year. Based on these assumptions, Table D.1 gives the total cost of monitoring a source (not including laboratory costs) based on a man day costing \$64.

Table D.1 TOTAL FIELD AND OFFICE COSTS

No. of outfalls	Manpower field costs	Manpower office costs	Equipment costs	Total cost
1 or 2	\$320	\$192	\$13	\$ 525
3 or 4	\$640	\$192	\$25	\$ 857
5 or 6	\$960	\$192	\$38	\$1190
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D.2 LABORATORY COST

The laboratory costs must include both the cost of making the analyses and the costs of report writing. If a private laboratory is used, then overhead costs will also be included. If the analyses are done by the monitoring agency or another government agency, then the capital equipment costs need not be included, since costs will exist regardless of the analyses made. For this project, a price list from a private laboratory was used to estimate the laboratory costs (see Table D.2).

REFERENCE

- D1. "Model Water Monitoring Program", Environmental Protection Agency, Office of Water Enforcement, 1974.

Table D.2 LABORATORY COSTS

Analysis	Cost	Analysis	Cost
Aluminum	\$ 8.50	Iron	\$ 7.50
Ammonia	10.00	Lead	7.50
BOD ₅	20.00	Manganese	7.50
Carbon	10.00	Mercury	15.00
COD	10.00	Nickel	7.50
Chloride	5.00	Nitrogen	10.00
Chloroform Extract	15.00	Oil-Grease	10.00
Chromium	7.50	pH	3.00
Coliforms-Total	15.00	Phenol	12.50
Coliforms-Fecal	15.00	Phosphorus	10.00
Copper	7.50	Dissolved Solids	10.00
Cyanide	15.00	Suspended Solids	5.00
Dissolved Oxygen	3.00	Tin	8.50
Fluoride	8.00	Zinc	7.50

APPENDIX E

BAYESIAN UPDATE FORMULA

Consider the case when both parameters of an independent normal process are estimated. Using the Bayesian approach, the parameters of the process, the mean μ and precision h , (the precision is equal $1/\sigma^2$ where σ is the standard deviation) are themselves treated as random variables. The most convenient [E1] joint distribution of the parameters - called the natural conjugate prior - is defined by

$$\begin{aligned} f_{NY}(\mu, h | m, n, v, \nu) \\ \propto \exp \left\{ -\frac{1}{2} h n (\mu - m)^2 \right\} h^{1/2} \\ \times \exp \left\{ -\frac{1}{2} \nu v h \right\} h^{\nu/2-1} \end{aligned} \quad (E.1)$$

This distribution is known as the normal-gamma distribution and is uniquely defined by the parameters m, n, v , and ν . m is the estimated mean of the process, v is the estimated variance of the process, n is a constant expressing the confidence (or uncertainty) in the estimated mean, and ν is a constant expressing the confidence in the estimated variance. For the case where the estimated mean, m , and variance, v , were obtained from N identically distributed, independent, normal random variables, $\{x_i\}$, using the sample mean and sample variance, that is

$$m = \frac{1}{N} \sum_{i=1}^N x_i$$

and

(E.2)

$$v = \frac{1}{N-1} \sum_{i=1}^N (x_i - m)^2$$

then n is equal N and v is equal $N-1$. Thus n and v express the degrees of freedom used to obtain the estimates m and v .

Suppose that estimates of the mean and variance, m and v , of a random process are available with confidence parameters n and v respectively. The prior distribution is normal-gamma with parameters (m, n, v, v) . If a new sample from the random process is made (independent from the process which yielded m and v) yielding a sufficient statistic (m', n', v', v') , then the posterior distribution is again normal-gamma with parameters [E1]

$$m'' = \frac{n'm' + nm}{n' + n} \quad (E.3a)$$

$$n'' = n' + n \quad (E.3b)$$

$$v'' = \frac{[v'v' + n'm'^2] + [vv + nm^2] - (n' + n)m''^2}{v' + v + 1} \quad (E.3c)$$

$$v'' = v' + v + 1 \quad (E.3d)$$

m'' and v'' are Bayesian posterior estimates of the mean and the variance and n'' and v'' are the corresponding confidence parameters. The formulas in (E.3) describe how to update old estimates (m, n, v, v) as new estimates (m', n', v', v') become available. If the new estimates are from a single data point z , then $m' = z$, $n' = 1$ and $v' = v' = 0$.

REFERENCE

- E1. Raiffa, H. and Schlaiffer, R., Applied Statistical Decision Theory,
The M.I.T. Press, Cambridge, Mass., 1961.

APPENDIX F
ESTIMATION OF THE BOD-DO AND COD-DO TRANSFER COEFFICIENTS
AND THE SATURATION LEVEL OF DO*

On streams, rivers, and vertically well-mixed reservoirs the maximum dissolved oxygen deficit ($D_{i, \text{BOD}}$) due to a BOD effluent is related to the BOD stream concentration at the effluent source ($C_{i, \text{BOD}}$), the BOD decay rate (K_d), the stream reaeration rate (K_a), and the waste dispersion rate. The initial BOD stream concentration is given by

$$C_{i, \text{BOD}} = \frac{M_{i, \text{BOD}} + C U_{i, \text{BOD}} Q U_i}{Q U_i + Q S_i} \quad (\text{F.1})$$

The relationship between $D_{i, \text{BOD}}$ and $C_{i, \text{BOD}}$ can be estimated using a transfer coefficient as

$$D_{i, \text{BOD}} = (K_{\text{BOD-DO}}) (C_{i, \text{BOD}}) \quad (\text{F.2})$$

$K_{\text{BOD-DO}}$ can be obtained using Figure F.1 along with Figure F.2. Figure F.1 shows that in streams, $K_{\text{BOD-DO}}$ varies primarily with K_a/K_d . Also, as rivers become more tidally influenced and broad, $K_{\text{BOD-DO}}$ increases. Values of K_a/K_d can be found for various applications using Figure F.2.

The damage due to COD loadings is difficult to quantify since there are many different kinds of chemicals, each with their own reaction

*Information derived from Simplified Mathematical Modeling; of Water Quality [F1]

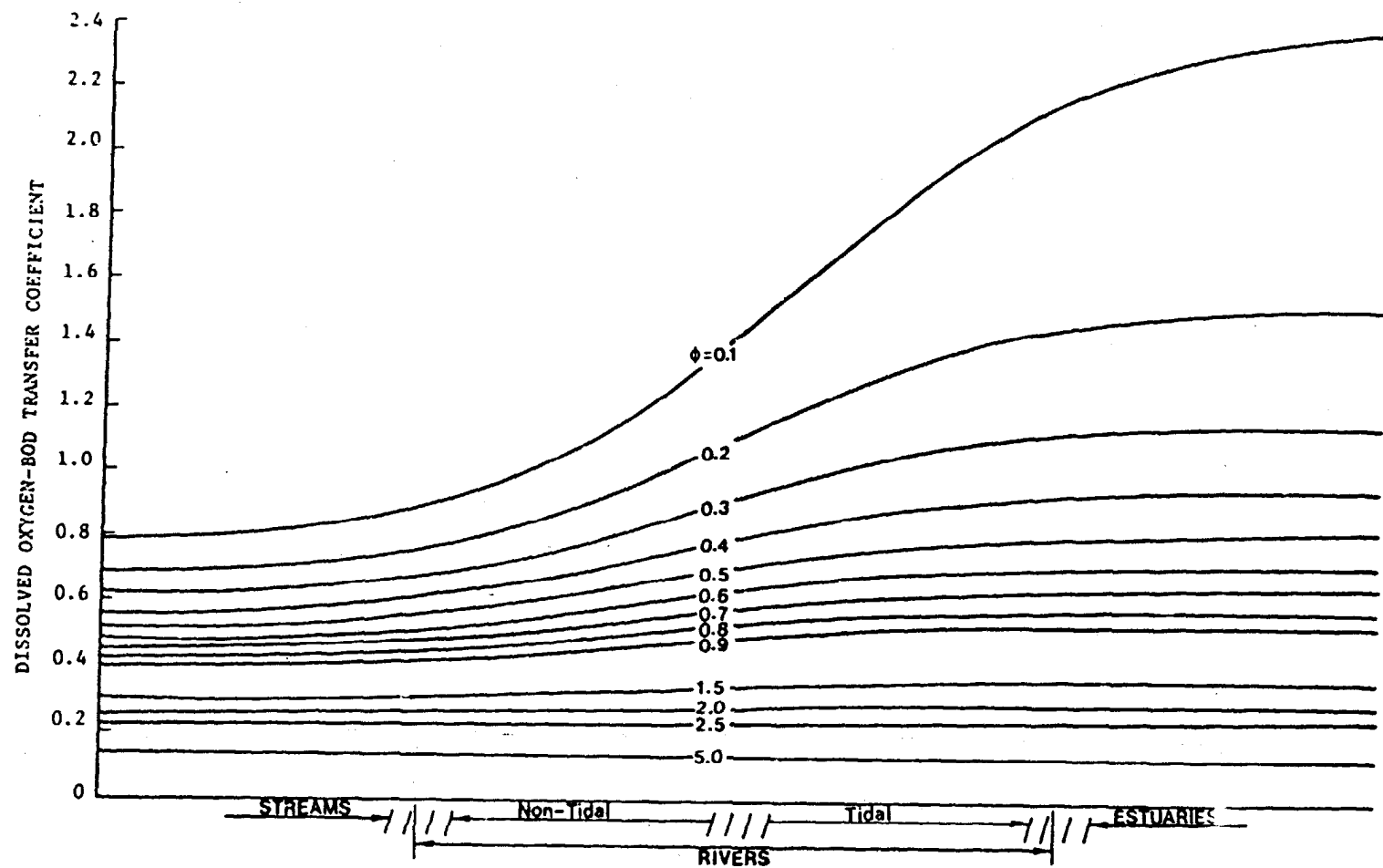


Figure F.1 Dissolved oxygen response as a function of water body type and ϕ .
 (Note: $\phi = K_a / K_d$)

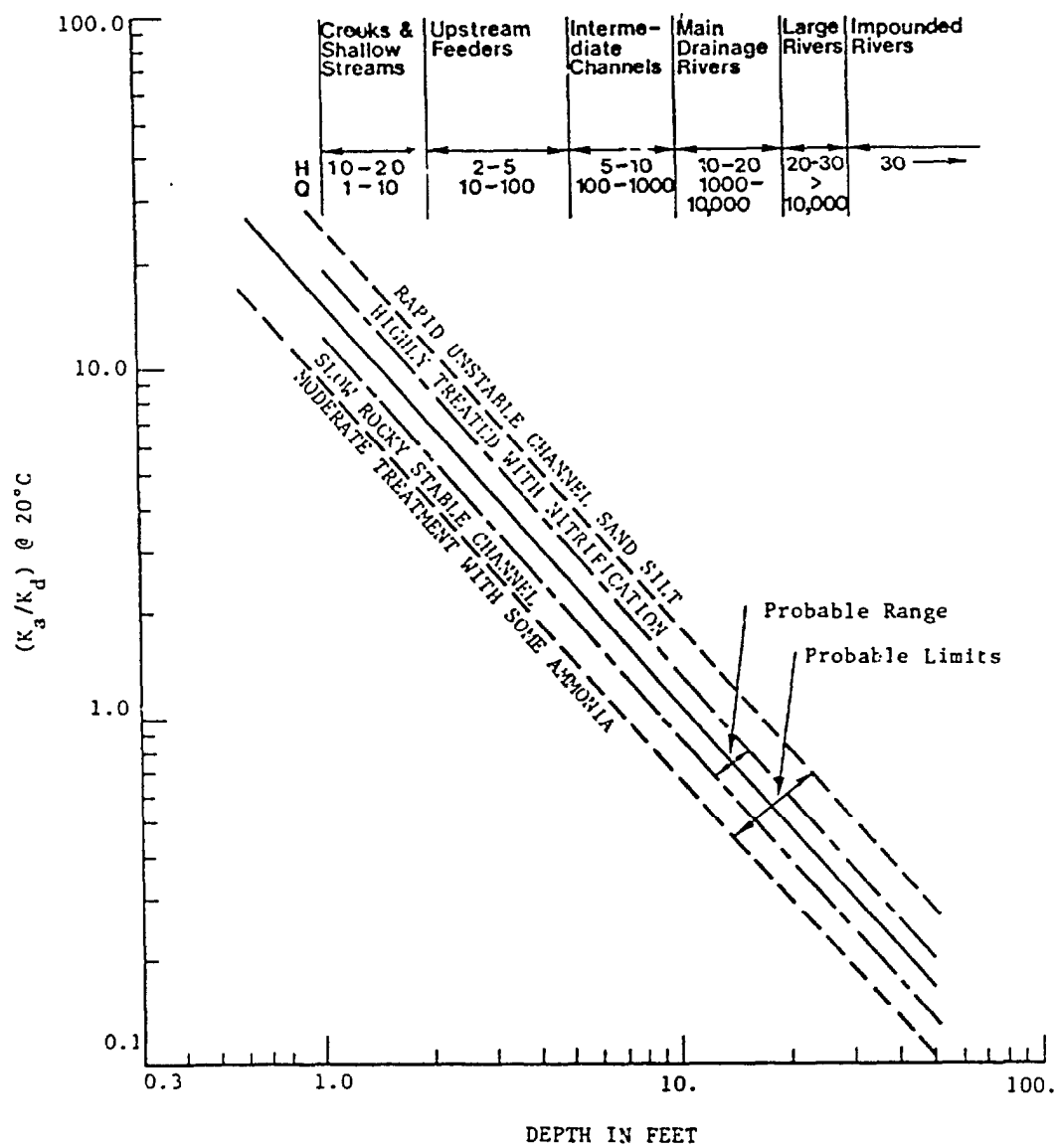


Figure F.2 (K_a/K_d) as a function of depth.

characteristics, which may demand oxygen. Furthermore, the lab tests for COD are performed by heating the sample, which probably would not indicate actual stream damages. For these reasons COD is generally not a modeled constituent and little is known about its stream characteristics. However, Prati et al. [F2] found that the damages due to COD are proportional to those from an equivalent concentration of BOD. The maximum DO uptake due to COD is related to the initial stream COD concentration (C_o) through a transfer function ($K_{\text{COD-DO}}$), which can be estimated as

$$K_{\text{COD-DO}} \approx 0.15 (K_{\text{BOD-DO}}). \quad (\text{F.3})$$

Therefore the COD transfer coefficient can be estimated using Figures F.1 and F.2 along with equation (F.3).

DOSAT, the saturation level of dissolved oxygen in the stream, can be found for various temperatures and salinities using Figure F.3.

REFERENCES

- F1. Hydrosience, Inc., Simplified Mathematical Modeling of Water Quality, Environmental Protection Agency, Washington, D.C., March 1971.
- F2. Prati, L., et al., "Assessment of Surface Water Quality by a Single Index of Pollution", Water Research (GB), Vol. 5, pp. 741-751, 1971.

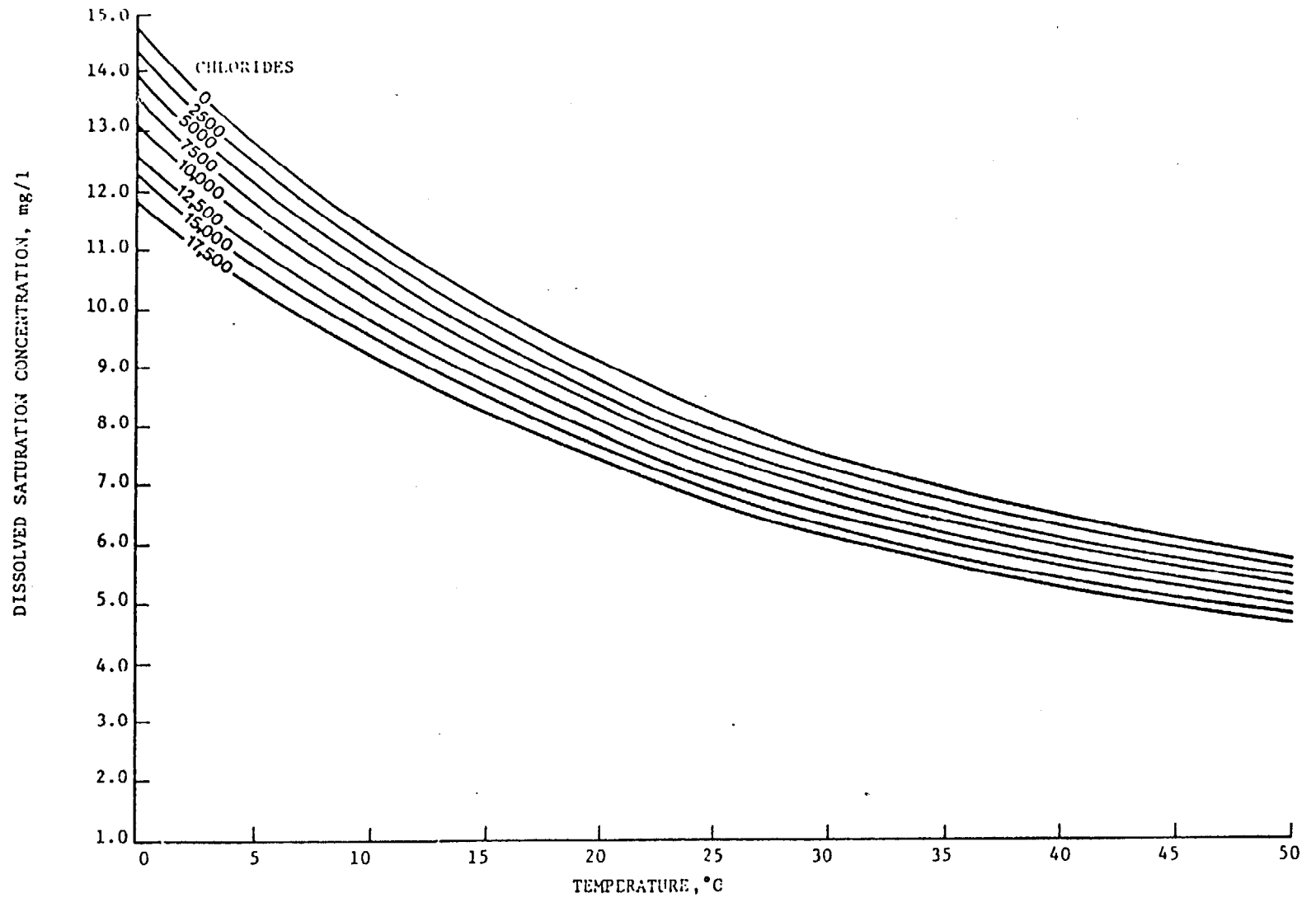


Figure F.3 Dissolved oxygen saturation versus temperature and chlorides.

APPENDIX G

DATA FOR DEMONSTRATION PROJECT

This appendix contains the statistical description, expected damage and probability of no violation for each constituent of each source for Case I and Case 11 of the demonstration project. The following notation is used:

DIST - Distribution

N - Normal

L - Lognormal

EST. MEAN - Estimated Mean

EST. SIGMA - Estimated Standard Deviation

The units for the standards for the various constituents are in kilograms, except for pH where the units are in pH. The units for estimated mean and estimated standard deviations are in kilograms, if the distribution is normal, and log kilograms if the distribution is lognormal (recall, in the lognormal case, the mean and standard deviation are of the logs of the data). For pH the units are in pH.

In the case of pH-max and pH-min only one value of expected damage and probability of violation is given since only one value is calculated (see Appendix C). Also note that the expected damage only appears once for each constituent of a source for the cases where there are two or more pipes from the source flowing into the stream.

G.1 DATA FOR CASE I

SOURCE 1

PIPE# 1 MEAN DISCHARGE (ML/DAY)= .3407 UPSTREAM FLOW (ML/DAY)= 4.8937

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROB. OF NO VIOLATION
PH-MAX	9.5000	N	7.5895	.6730	*****	*****
PH-MIN	6.5000	N	7.5895	.4395	.2860	.9911
CHROMIUM	.5299	L	-1.9113	.8876	.8077	.9673
NICKEL	2.6497	L	-1.4006	.6635	1.3251	.9970
CHLOROFORM EXTRACT	3.9746	L	-1.7560	1.6513	1.8245	.9233

PIPE# 2 MEAN DISCHARGE (ML/DAY)= .0151 UPSTREAM FLOW (ML/DAY)= 4.8937

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROB. OF NO VIOLATION
BOD5	.1987	N	.1184	.0484	1.2426	.9515
SUSPENDED SOLIDS	.2650	N	.2010	.0896	.0038	.7624
CHLORIDE	2.6500	N	.0306	.0044	.0005	1.0000

SOURCE EXPECTED DAMAGE 1.8245
SOURCE PROBABILITY OF NO VIOLATION .6404

SOURCE 2

PIPE# 1 MEAN DISCHARGE (ML/DAY)= .3779 UPSTREAM FLOW (ML/DAY)= 185.9600

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROB. OF NO VIOLATION
PHOSPHORUS	.8025	L	-1.3866	.5414	.0931	.9914
PH-MAX	9.5000	N	7.1500	.5119	*****	*****
PH-MIN	6.5000	N	7.1500	.4994	.0104	.9035
SUSPENDED SOLIDS	24.9476	L	.7711	.3048	.0502	.9800
CHLOROFORM EXTRACT	6.0186	L	.0432	.2269	1.3554	.9994

PIPE# 2 MEAN DISCHARGE (ML/DAY)= .7621 UPSTREAM FLOW (ML/DAY)= 185.9600

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROB. OF NO VIOLATION
PHOSPHORUS	.9388	L	-.2520	.3547	*****	.7367
PH-MAX	9.5000	N	7.9333	.3064	*****	*****
PH-MIN	6.5000	N	7.9200	.2460	*****	1.0000
SUSPENDED SOLIDS	58.9670	L	1.5876	.5508	*****	.6302
CHLOROFORM EXTRACT	7.0407	L	.5393	.2396	*****	.9009

SOURCE EXPECTED DAMAGE 1.3554
SOURCE PROBABILITY OF NO VIOLATION .3669

 SOURCE 3

PIPE# 1	MEAN DISCHARGE (ML/DAY)=	.0750	UPSTREAM FLOW (ML/DAY)=	1891.4000		
CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROB. OF NO VIOLATION
PH=MAX	9.5000	N	7.7258	.7820	*****	*****
PH=MIN	6.5000	N	7.7258	.5798	.0005	.9711
SUSPENDED SOLIDS	16.0875	N	.5263	.4465	.0000	1.0000
PHOSPHORUS	3.2175	N	.0477	.0704	.0006	1.0000

 SOURCE EXPECTED DAMAGE .0006
 SOURCE PROBABILITY OF NO VIOLATION .9711

SOURCE 4

PIPE= 1 MEAN DISCHARGE (ML/DAY)= .8026 UPSTREAM FLOW (ML/DAY)= 51.3840

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PRGR. OF NO VIOLATION
PH-MAX	9.0000	N	8.0385	.1256	*****	*****
PH-MIN	6.0000	N	8.0385	.1234	.0451	1.0000
SUSPENDED SOLIDS	18.9265	L	.7329	.1648	.0263	.9995
CHLOROFORM EXTRACT	7.5706	L	.5695	.1684	3.4284	.9670

PIPE= 2 MEAN DISCHARGE (ML/DAY)= .3762 UPSTREAM FLOW (ML/DAY)= 51.3840

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PRGR. OF NO VIOLATION
PH-MAX	9.0000	N	7.9083	.2509	*****	*****
PH-MIN	6.0000	N	7.9833	.2342	*****	1.0000
SUSPENDED SOLIDS	7.5705	L	.8516	.2116	*****	.5517
CHLOROFORM EXTRACT	3.0282	L	.2921	.2107	*****	.8153

SOURCE EXPECTED DAMAGE 3.4284
SOURCE PROBABILITY OF NO VIOLATION .4348

SOURCE 6

PIPE# 1 MEAN DISCHARGE (ML/DAY)= 18.7919 UPSTREAM FLOW (ML/DAY)= 1358.0000

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROM. OF NO VIOLATION
PH-MAX	10.3000	N	7.6797	.4289	*****	*****
PH-MIN	5.8000	N	7.6797	.3663	.0418	1.0000
OIL-GREASE	251.8800	N	140.9278	128.6569	4.0479	1.0000
PHENOL	.9072	L	-.9000	.3921	1.4198	.9904

PIPE# 2 MEAN DISCHARGE (ML/DAY)= 34.2089 UPSTREAM FLOW (ML/DAY)= 1358.0000

CONSTITUENT	STANDARD	DIST	EST. MEAN	EST. SIGMA	EXPECTED DAMAGE	PROM. OF NO VIOLATION
PH-MAX	10.3000	N	7.8392	.2907	*****	*****
PH-MIN	5.8000	N	7.8392	.3918	*****	1.0000
OIL-GREASE	459.2050	N	165.8726	68.0088	*****	1.0000
PHENOL	1.3608	L	-.6423	.3921	*****	.9762

SOURCE EXPECTED DAMAGE 4.0479
SOURCE PROBABILITY OF NO VIOLATION .9668
